

## On a regularization technique for Kovarik-like approximate orthogonalization algorithms

AURELIAN NICOLA, CONSTANTIN POPA and ULRICH RÜDE

### ABSTRACT.

In this paper we consider four versions of Kovarik's iterative orthogonalization algorithm, for approximating the minimal norm solution of symmetric least squares problems. Although the theoretical convergence rate of these algorithms is at least linear, in practical applications we observed that a too big number of iterations can dramatically deteriorate the already obtained approximation. In this respect we analyse the above mentioned Kovarik-like methods according to the modifications they make on the "machine zero" eigenvalues of the problem's (symmetric) matrix. We establish a theoretical almost optimal formula for the number of iterations necessary to obtain an enough accurate approximation, as well as to avoid the above mentioned troubles. Experiments on collocation discretization of a Fredholm first kind integral equation illustrate the efficiency of our considerations.

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FACULTY OF MATHEMATICS AND COMPUTER SCIENCE  
"OVIDIUS" UNIVERSITY  
900527, CONSTANTA, ROMANIA  
E-mail address: [anicola@univ-ovidius.ro](mailto:anicola@univ-ovidius.ro), [cpopa@univ-ovidius.ro](mailto:cpopa@univ-ovidius.ro)

INSTITUTE FOR COMPUTER SCIENCE 10 (SYSTEMSIMULATION),  
FRIEDRICH-ALEXANDER-UNIVERSITÄT ERLANGEN-NÜRNBERG,  
GERMANY  
E-mail address: [ulrich.ruede@informatik.uni-erlangen.de](mailto:ulrich.ruede@informatik.uni-erlangen.de)